Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 22: Variation

Definition 1. **<u>Direct Variation:</u>** *A quantity y is said to* ***vary directly*** *as the quantity x, or y is* ***directly proportional*** *to x, if there is a constant k such that $\underline{y = kx}$. The constant $k \neq 0$ is called the* ***constant of variation*** *or the* ***constant of proportionality****.*

Example 1. *Suppose y varies directly as x. If y is 6 when x is 30, find y when $x = 180$.*

<u>Solution:</u> *Since y varies directly as x, then $y = kx$. Moreover, $y = 6$ when $x = 30$ implies that $6 = k(30)$, then $k = \frac{6}{30} = \frac{1}{5}$. Now, to find y when $x = 120$: we put $x = 120$ and $k = \frac{1}{5}$ into $y = kx$.*

$$\begin{aligned} y &= kx \\ &= (\frac{1}{5})(120) \\ &= 24. \end{aligned}$$

Definition 2. **<u>Direct Variation with Powers:</u>** *A quantity y* ***varies directly*** *as the nth power of x means that*

$$y = kx^n,$$

where $k \neq 0$ and $n > 0$.

Example 2. *Suppose that the volume V of a sphere varies directly as the cube of its radius r. If V is 972π when r is 9, find V when $r = 6$.*

<u>Solution:</u> *The volume V of a sphere varies directly as the cube of its radius r implies that $V = kr^3$. Given that V is 972π when r is 9 implies that $972\pi = k(9)^3$. So $k = \frac{4\pi}{3}$. To find V when $r = 6$,*

$$\begin{aligned} V &= kr^3 \\ &= (\frac{4\pi}{3})(6)^3 \\ &= 288\pi. \end{aligned}$$

Definition 3. **<u>Inverse Variation:</u>** *A quantity y is said to* ***vary inversely*** *as the quantity x, or y is* ***inversely proportional*** *to x, if*

$$y = \frac{k}{x},$$

where k is a nonzero constant.

Example 3. *Suppose y varies inversely as x and $y = 35$ when $x = 11$. Find y when $x = 55$.*

<u>Solution:</u> *Since y varies inversely as x, then $y = \frac{k}{x}$. Given that $y = 35$ when $x = 11$ implies that $35 = \frac{k}{11}$. Thus, $k = (35)(11) = 385$. To find y when $x = 55$,*

$$\begin{aligned} y &= \frac{k}{x} \\ &= \frac{385}{55} \\ &= 7. \end{aligned}$$

Definition 4. **Inverse Variation with Powers:** *A quantity y **varies inversely** as the nth power of x means that*

$$y = \frac{k}{x^n},$$

where $k \neq 0$ and $n > 0$.

Example 4. *The intensity I of illumination from a light source is inversely proportional to the square of the distance d from the source. Suppose that the intensity is 320 candlepower at a distance of 10 feet from a light source.*

- **a.** *What is the intensity at 5 feet from the source?*
- **b.** *How far away from the source will the intensity be 400 candlepower?*

Definition 5. **Joint and Combined Variation** *The expression z **varies jointly** as x and y **means** that $z = kxy$ for some nonzero constant k.*

*Similarly, if m and n are positive numbers, then the expression z **varies jointly** as the nth power of x and mth power of y **means** that $z = kx^n y^m$ for some nonzero constant k.*

Example 5. *Suppose that z varies directly as the square root of x and inversely as the square of y. Let $z = 24$ when $x = 16$ and $y = 3$. Find x when $z = 27$ and $y = 2$.*

Solution: *Since z varies directly as the square root of x and inversely as the square of y, then $z = k\frac{\sqrt{x}}{y^2}$. Since $z = 24$ when $x = 16$ and $y = 3$, then*

$$\begin{aligned} z &= k\frac{\sqrt{x}}{y^2} \\ 24 &= k\frac{\sqrt{16}}{(3)^2} \\ (24)(9) &= k(\sqrt{16}) \\ 216 &= k(4). \end{aligned}$$

Then $k = 54$. To find x when $z = 27$ and $y = 2$, then

$$\begin{aligned} z &= k\frac{\sqrt{x}}{y^2} \\ 27 &= 54\frac{\sqrt{x}}{(2)^2} \\ 27 &= 54\frac{\sqrt{x}}{4} \\ (27)(4) &= 54(\sqrt{x}) \\ 108 &= 54(\sqrt{x}) \end{aligned}$$

Then $\sqrt{x} = \frac{108}{54} = 2$ which implies that $x = 4$.